Workshop Program 'Rational Points 2019'

Sunday, July 14

	Arrival
from 14:30	*Reception is open*
from 15:00	*Coffee and Snacks can be bought in the restaurant*
18:30–21:00 (ca.)	*Dinner*

Monday, July 15

08:00–09:15	*Breakfast*
09:20–09:30	Michael Stoll: *Opening and Welcome*
09:30–10:30	Rachel Newton: **A walk on the wild side: evaluating p-torsion Brauer classes at p-adic points (I)**
10:30–11:00	*Coffee*
11:00–11:30	Martin Bright: **A walk on the wild side: evaluating p-torsion Brauer classes at p-adic points (II)**
11:45–12:15	Dan Loughran: **Brauer-Manin obstruction for the Erdős-Straus equation**
12:30–13:30	*Lunch*
15:00–16:00	*Coffee*
16:00–16:45	Alexei Skorobogatov: **Cohomology and the Brauer groups of diagonal surfaces**
17:00–17:30	Jean-Louis Colliot-Thélène: **Variations sur un thème d'André Néron**
17:45–18:15	Christopher Frei: **Abelian extensions with prescribed norms**
18:30–19:30	*Dinner*
20:00–?	**Open Problems**

Tuesday, July 16

08:00–09:15	*Breakfast*
09:30–10:30	Jennifer Balakrishnan: **Chabauty-Coleman experiments for genus 3 hyperelliptic curves**
10:30–11:15	*Coffee*
11:15–12:15	Bas Edixhoven and Guido Lido: **Geometric quadratic Chabauty**
12:30–13:30	*Lunch*
15:00–16:00	*Coffee*
16:00–16:45	Netan Dogra: **Quadratic Chabauty and modular curves**
17:00–17:30	Steffen Müller: **Computational aspects of quadratic Chabauty**
17:45–18:15	David Holmes: **Explicit arithmetic intersection theory and computation of Néron-Tate heights**
18:30–19:30	*Dinner*
20:00–?	**Reinventing "Rational Points"?** Discussion on the future of the workshop series

Wednesday, July 17

08:00–09:15	*Breakfast*
09:30–10:30	Ari Shnidman: **Monogenic cubic fields**
10:30–11:15	*Coffee*
11:15–12:15	Olivier Wittenberg: **Tight approximation for rational points over function fields**
12:30–13:30	*Lunch*
	Free Afternoon, *Coffee* and *Dinner* on request

Thursday, July 18

08:00–09:15	*Breakfast*
09:30–10:30	David Zureick-Brown: **Mazur's Program B**
10:30–11:15	*Coffee*
11:15–12:15	Wei Ho: **Integral points on elliptic curves**
12:30–13:30	*Lunch*
15:00–16:00	*Coffee*
16:00–16:45	Nils Bruin: **Quasi-hyperbolicity of nodal surfaces**
17:00–17:30	Filip Najman: **Modularity of elliptic curves over totally real cubic fields**
17:45–18:15	Tom Fisher: **13-congruences of elliptic curves**
18:30–19:30	*Dinner*
20:00–?	**Software Demonstrations:** Nils Bruin: **Numerical computation of endomorphism rings** Stefan Wewers: **Models of curves over local fields** Drew Sutherland: **LMFDB News** Michael Stoll: **Minimization and reduction of plane curves**

Friday, July 19

08:00–09:15	*Breakfast*
09:30–10:30	Samir Siksek: **Efficient resolution of Thue–Mahler equations**
10:30–11:15	*Coffee*
11:15–12:15	Brian Lawrence: **Toward algorithmic Mordell**
12:30–13:30	*Lunch*
15:00–16:00	*Coffee*
16:00–16:45	Marta Pieropan: **Campana points on toric varieties**
17:00–17:30	Ulrich Derenthal: **On Manin's conjecture for certain smooth spherical Fano varieties**
17:45–18:15	Ronald van Luijk: **Verifying Zariski density of rational points on del Pezzo surfaces of degree 1**
18:30–19:30	*Dinner*

Saturday, July 20

07:30–09:30	*Breakfast*
	Departure

Abstracts

Rachel Newton and Martin Bright:
A walk on the wild side: evaluating p-torsion Brauer classes at p-adic points

In order to compute Brauer-Manin obstructions to the Hasse principle and weak approximation, one must evaluate elements of the Brauer group at adelic points of a variety X. If an element of the Brauer group has order coprime to p, then its evaluation at a p-adic point factors via reduction of the point modulo p. For p-torsion elements this is no longer the case: in order to compute the evaluation map one must know the point to a higher p-adic precision. Classifying Brauer group elements according to the precision required to evaluate them at p-adic points gives a filtration on $\mathrm{Br}\, X[p]$ *which we describe using work of Bloch and Kato.*

Dan Loughran: **Brauer-Manin obstruction for the Erdős-Straus equation**

The Erdős-Straus conjecture states that $4/n$ can be written as the sum of three Egyptian fractions, for any integer $n > 1$. In this talk I explain what the Brauer-Manin obstruction has to say about this problem. This is joint work with Martin Bright.

Alexei Skorobogatov: **Cohomology and the Brauer groups of diagonal surfaces**

The cohomology of a Fermat surface has an explicit description which gives the Hodge structure and the intersection form in terms of the action of roots of unity on the coordinates. The Galois representation over the cyclotomic field can also be explicitly described in terms of Jacobi sums, following Weil. I will explain how to compute the full Brauer group of an arbitrary diagonal surface from this data. (Joint work with D. Gvirtz.)

Jean-Louis Colliot-Thélène: **Variations sur un thème d'André Néron**

Let $X \to U$ be a family of abelian varieties over a number field k. If the total space X is dominated by a variety which satisfies weak weak approximation, for example if X is unirational over k, then the set of rational points of U whose fibre has Mordell-Weil rank strictly bigger than that of the generic fibre is dense for the Zariski topology on U.

Christopher Frei: **Abelian extensions with prescribed norms**

We discuss quantitative and constructive results concerning extensions K/k of a given number field k, with given abelian Galois group G, in which finitely many given elements of k can be realised simultaneously as norms from K. This is joint work with Dan Loughran and Rachel Newton, and with Rodolphe Richard.

Jennifer Balakrishnan: **Chabauty-Coleman experiments for genus 3 hyperelliptic curves**

We describe a Chabauty–Coleman computation of rational points on genus 3 hyperelliptic curves defined over the rationals whose Jacobians have Mordell–Weil rank 1, which was carried out on approximately 17,000 curves from a forthcoming database of genus 3 hyperelliptic curves. We discuss a few surprising examples where the zero locus includes global points not found in the set of rational points. This is joint work with Francesca Bianchi, Victoria Cantoral-Farfán, Mirela Çiperiani and Anastassia Etropolski.

Bas Edixhoven and Guido Lido: **Geometric quadratic Chabauty**

Joint work with Guido Lido. Chabauty's method to find all rational points on a curve C *over* $\mathbb{Q}$ *of genus* $g > 1$ *is to intersect, for a suitable prime* p*, inside the* p*-adic Lie group* $J(\mathbb{Q}_p)$ *(with* J *the jacobian of* C*), the 1-dimensional* p*-adic manifold* $C(\mathbb{Q}_p)$ *with the closure of* $J(\mathbb{Q})$*. This closure is a* p*-adic Lie group of dimension at most* r*, the rank of* $J(\mathbb{Q})$*. If* $r < g$ *then this works well. Minhyong Kim has a program called 'nonabelian Chabauty', where deeper quotients of the fundamental group of* C *are exploited (*J *corresponds to the abelianisation). The recently developed 'quadratic Chabauty method' (Balakrishnan, Dogra, Muller, Tuitman, Vonk) can treat cases where* r *is larger and* J *has sufficiently many symmetric endomorphisms, notably the 'cursed curve'. In this lecture I will report on a geometric approach to the quadratic Chabauty method in terms of the Poincaré torsor on* J *times its dual: the geometric setup, a bound (under favorable conditions) on the number of rational points, how to compute everything, and a genus 2 example by Guido Lido.*

Netan Dogra: **Quadratic Chabauty and modular curves**

When applying Chabauty's method to determine the rational points on a curve it is enough to find an isogeny factor of the Jacobian with small Mordell-Weil rank. I will describe an analogue of this in the context of Quadratic Chabauty, and how this can be used to prove finiteness of the quadratic Chabauty set when the curve is a (hyperbolic) non-split Cartan modular curve or Atkin-Lehner quotient of $X_0(N)$*. This is joint work with Samuel Le Fourn.*

Steffen Müller: **Computational aspects of quadratic Chabauty**

Quadratic Chabauty is a method to compute the rational points on a curve that satisfies some restrictive conditions. In this talk I will discuss how to carry it out in practice and how to remove some of the conditions. I will also mention some new examples of modular curves whose rational points were computed using this approach. This is joint work with Jennifer Balakrishnan, Netan Dogra, Jan Tuitman and Jan Vonk.

David Holmes: **Explicit arithmetic intersection theory and computation of Néron-Tate heights**

We describe a simple procedure to compute the intersection pairing between two divisors meeting properly on a regular arithmetic surface. Putting this together with recent advances in the computation of various integrals on Riemann surfaces allows us to give a general algorithm to compute the Néron-Tate height pairing between points on Jacobians, and thence to compute regulators up to integral squares. Our algorithm is implemented in MAGMA, and has been applied to smooth plane quartics (extending previous work in the hyperelliptic case). This is joint work with Steffen Mueller and Raymond van Bommel.

Ari Shnidman: **Monogenic cubic fields**

A cubic field is monogenic if its ring of integers is generated, as a ring, by a single element. I'll describe work with Alpoge and Bhargava where we study the question: what proportion of cubic fields are not monogenic, when ordered by discriminant? There are local obstructions which make it fairly easy to see that the answer is at least $> 0\%$*. We show that even among those cubic fields which have no local obstructions, a positive proportion are indeed not monogenic. If we restrict to thinner families, such as cyclic cubics or pure cubics, we are able to prove that 100% are not monogenic. The proofs involve studying Selmer groups in various families of elliptic curves with* j*-invariant 0.*

Olivier Wittenberg: **Tight approximation for rational points over function fields**

(Joint work with Olivier Benoist.) The notion of weak approximation plays a central rôle in the study of rational points of rationally connected varieties, both when the ground field is a number field and when it is the function field of a complex or real curve. In the latter situation, we introduce a stronger notion, "tight approximation", and establish descent and fibration theorems for it. We draw consequences, on the one hand, for weak approximation, and, on the other hand, for the algebraicity of the homology of the real locus of some real varieties.

David Zureick-Brown: **Mazur's Program B**

I'll discuss recent progress on Mazur's "Program B" — the problem of classifying all possibilities for the "image of Galois" for an elliptic curve over $\mathbb{Q}$ (equivalently, classification of all rational points on certain modular curves X_H).

This will include my own recent work with Jeremy Rouse which completely classifies the possibilities for the 2-adic image of Galois associated to an elliptic curve over the rationals, and work in progress for other prime powers. I will also survey other very recent results by many authors.

Wei Ho: **Integral points on elliptic curves**

We show that the second moment for the number of integral points on elliptic curves over $\mathbb{Q}$ is bounded. The main new ingredient in our proof is an upper bound on the number of integral points on an affine integral Weierstrass model of an elliptic curve depending only on the rank of the curve and the square divisors of the discriminant. We obtain the bound by studying a bijection first observed by Mordell between integral points on these curves and certain types of binary quartic forms. The results on moments then follow from Hölder's inequality, analytic techniques, and results on bounds on the average sizes of Selmer groups in the families. This is joint work with Levent Alpoge.

Nils Bruin: **Quasi-hyperbolicity of nodal surfaces**

A surface is called algebraically quasi-hyperbolic if it only contains finitely many curves of genus zero or one. One way to show that a projective surface is quasi-hyperbolic is to show that the m-th symmetric power of the sheaf of differentials on the surface has global sections. Various results on hyperbolicity of sufficiently general surfaces in certain families have been obtained this way.

Bogomolov and Oliveira observed that the presence of nodal singularities on a hypersurface contributes to the existence of m-th symmetric differentials on the desingularization. We refine and generalize their results to complete intersections, giving bounds rather than asymptotic results. We also outline a way to explicitly determine a locus containing all the non-hyperbolic curves on the surface.

As an application, we show that any genus zero curve on the so-called rational cuboid surface needs to pass through singularities of the surface (whether the surface is quasi-hyperbolic remains open).

This is joint work with Anthony Várilly-Alvarado.

Filip Najman: **Modularity of elliptic curves over totally real cubic fields**

We will prove that all elliptic curves over totally real cubic fields are modular and explain the ingredients that go into this proof. This is joint work with Maarten Derickx and Samir Siksek.

Tom Fisher: **13-congruences of elliptic curves**

Elliptic curves are said to be n-congruent if their n-torsion subgroups are isomorphic as Galois modules. Frey and Mazur conjectured that if elliptic curves over $\mathbb{Q}$ are n-congruent for n sufficiently large, then they must be isogenous. I will report on work exhibiting non-trivial pairs of 13-congruent elliptic curves over $\mathbb{Q}$. One reason why the case $n = 13$ is interesting is that it is the smallest value of n for which all the surfaces parametrising pairs of n-congruent elliptic curves are of general type. I have computed equations for these surfaces as a double cover of the plane, ramified over (a highly singular model of) Baran's modular curve of level 13.

Samir Siksek: **Efficient resolution of Thue–Mahler equations**

A Thue–Mahler equation has the form

$$F(X, Y) = p_1^{z_1} \cdots p_r^{z_r}$$

where F is an irreducible homogeneous binary form of degree at least 3 with integer coefficients, and $p_1, \dots, p_r$ are primes. A standard algorithm due to Tzanakis and de Weger solves Thue–Mahler equations when the degree and the number of primes is small. We give lattice-based sieving techniques that are capable of handling large Thue–Mahler equations. This is joint work with Adela Gherga, Rafael von Känel and Benjamin Matschke.

Brian Lawrence: **Toward algorithmic Mordell**

I will outline a proof of Mordell's conjecture / Faltings's theorem using p-adic Hodge theory. Joint with Akshay Venkatesh.

Marta Pieropan: **Campana points on toric varieties**

We call Campana points an arithmetic notion of points on Campana's orbifolds that has been first studied by Campana and Abramovich, and that interpolates between the notions of rational and integral points. This talk reports on work in progress with Damaris Schindler on the universal torsor method for Campana points of bounded height on toric varieties.

Ulrich Derenthal: **On Manin's conjecture for certain smooth spherical Fano varieties**

We discuss a proof of Manin's conjecture for certain spherical Fano varieties. This is joint work in progress with Valentin Blomer, Jörg Brüdern and Giuliano Gagliardi.

Ronald van Luijk: **Verifying Zariski density of rational points on del Pezzo surfaces of degree 1**

Let S be a del Pezzo surface of degree 1 over a number field k. The main goal of this talk is to give easily verifiable sufficient conditions under which its set $S(k)$ of rational points is Zariski dense. It is well-known that almost all fibers of the anticanonical map $\varphi\colon S \dashrightarrow \mathbb{P}^1$ are elliptic curves with the unique base point of φ as zero. Suppose that $P \in S(k)$ is a point of finite order $n > 1$ on its fiber. Then there is another elliptic fibration on the blow-up of S at P. We will see where it comes from and how it can be used to define a proper closed subset $Z \subset S$ such that (1) it is easy to verify for any point on S whether it lies in Z, and (2) the set $S(k)$ contains a point outside Z if and only if $S(k)$ is Zariski dense. In other words, if $S(k)$ is Zariski dense, then we can prove this by exhibiting a rational point outside Z. We will also compare this to previous work. This is joint work with Jelle Bulthuis inspired by an example of Noam Elkies.