Isogenies over quadratic fields of elliptic curves with rational j-invariant

Borna Vukorepa

University of Zagreb

bovukor@math.hr

For a number field K, it is well known that elliptic curves E/K with a cyclic K-rational n-isogeny are parametrized (up to an isomorphism over $\overline{K}$) by the K-rational points on the modular curve $X_0(n)$. Bruin and Najman, Ozman and Siksek, and Box described all the quadratic points on the modular curves of genus $2 \leq g(X_0(n)) \leq 5$.

That information tells us something about the elliptic curves and isogenies defined over quadratic fields. It is therefore reasonable to ask ourselves a slightly different question: if E has a rational j-invariant, what are the possible degrees of a cyclic isogeny of E defined over a quadratic field?

This question is closely connected to the question of possible images of the mod n Galois representation of an elliptic curve E defined over $\mathbb{Q}$. It was proved by Najman that if E is a non-CM elliptic curve with a rational j-invariant with an isogeny of prime degree p, then $p \in \{2, 3, 5, 7, 11, 13, 17, 37\}$ as long as the degree of the field of definition of the isogeny is at most 7.

Motivated by that work, we will answer the same question for quadratic number fields and for all possible cyclic isogeny degrees (not only prime degrees).