Abstract

Finding simple 8-designs with a probabilistic algorithm

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E. Kramer and D. Mesner developed a very successful approach to construct simple t-designs: Prescribe a group of automorphisms, construct the incidence matrix of the orbits and solve the resulting system of linear Diophantine equations. Finding $\{0, 1\}$-solutions of systems of linear Diophantine equations is an NP-hard problem and forms still the bottleneck of this approach. In a series of papers coauthored by the first and the last author an exhaustive search algorithm based on lattice basis reduction has been used to construct 7-, 8- and 9-designs. Here, a new probabilistic algorithm by Schnorr called random sampling was implemented and enabled the construction of new 8-$(40, 12, \lambda)$ designs for $\lambda \in \{i \cdot 3240, i \cdot 3240 + 320 \mid i = 2, 3, 4\}$ with automorphism group $\mathrm{PSL}(4, 3)$.