Abstract

On intersections of perfect binary codes

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A perfect 1-error correcting binary code is a subset $C$ of the direct product $E^n$ of $n$ copies of the finite field $E$ with two elements satisfying the following condition: for any word $x \in E^n$ there is a unique word $c \in C$ such that the number of coordinates in which $x$ and $c$ differ is at most one.

Here we are concerned with the following problem: which are the possibilities for the number of words $\eta(C_1, C_2)$ in the intersection of two perfect codes $C_1$ and $C_2$, containing the all-zero word? This problem was proposed by Etzion and Vardy in 1998. They established that for any two distinct perfect codes $C_1$ and $C_2$ of length $n = 2^m - 1$

$$2 \leq \eta(C_1, C_2) \leq 2^{n-\log_2(n+1)} - 2^{(n-1)/2}.$$ They also proved that there are perfect codes $C_1$ and $C_2$ of length $n = 2^m - 1$, for $m \geq 3$, such that

$$\eta(C_1, C_2) = k2^{(n-1)/2} \text{ for all } k = 1, 2, \ldots, 2^{(n+1)/2-\log_2(n+1)} - 1$$

and constructed pairs of perfect codes $C_1$ and $C_2$ with $\eta(C_1, C_2) = 2$ for any admissible length $n$.

We prove that for any two integers $k_1$ and $k_2$ satisfying

$$1 \leq k_i \leq 2^{(n+1)/2-\log_2(n+1)}, \quad i = 1, 2,$$

there exist perfect codes $C_1$ and $C_2$, both of length $n = 2^m - 1$, $m \geq 4$, with intersection number

$$\eta(C_1, C_2) = 2k_1k_2.$$