Abstract

Line spreads of polar spaces of rank 4 inducing generalized quadrangles

Harm Pralle
Institut Computational Mathematics, TU Braunschweig,
Pockelsstr. 14, 38106 Braunschweig

Let $\Pi$ be a polar space of rank 4 over a field $K$ such that the generalized quadrangle $Res^+(\alpha)$ of a line $\alpha$ of $\Pi$ which consists of the planes and 3-spaces of $\Pi$ containing $\alpha$, admits a spread. Let $\mathcal{L}$ be a line-spread of $\Pi$ with the following property:

Let $\mathcal{D}$ be the set of 3-spaces of $\Pi$ in which $\mathcal{L}$ induces spreads. For every point $\Sigma$ of $\Pi$, the 3-spaces of $\mathcal{D}$ containing $\Sigma$ all contain the spread line $\lambda \in \mathcal{L}$ covering $\Sigma$ and form a spread of the generalized quadrangle $Res^+(\lambda)$.

Given such a spread $\mathcal{L}$, we show that $\Gamma = (\mathcal{L}, \mathcal{D})$ is a generalized quadrangle which we characterize for the classical polar spaces $\Pi \cong Sp_4(K)$ and $O^-_{10}(K)$ as $Sp_4(K(\zeta))$ and $H_5(K(\zeta))$, respectively, where $K(\zeta)$ is a quadratic field extension of $K$. For finite polar spaces, we show they are the only two admitting such a spread. We give an example of a spread $\mathcal{L}$ for the infinite hermitian polar space $H_8(C)$ over the complex numbers $C$ where $\Gamma = (\mathcal{L}, \mathcal{D})$ is a hermitian generalized quadrangle $H_4(Q)$ over the quaternions.

This research is motivated by the following: Dualizing $\Pi$, the point set $\bigcup_{X \in \mathcal{D}} X^\perp$ of the dual polar space $\Delta$ dual of $\Pi$ is a hyperplane of $\Delta$ intersecting each symp $\Sigma$, i.e. an element of maximal type of $\Delta$, in the neighbours of an ovoid of a quad of $\Sigma$. 

1