Abstract

Nonexistence of Perfect Steiner Triple Systems of Orders 19 and 21

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A *Steiner triple system* of order v is a pair $(V, \mathcal{B})$, where V is a set of v *points* and $\mathcal{B}$ is a set of 3-subsets of V—called *blocks*—such that every 2-subset of V occurs in a unique block. For every 2-subset $\{x, y\} \subseteq V$ and the associated block $\{x, y, z\} \in \mathcal{B}$, the *cycle graph* G_{xy} is the graph with vertex set $V \setminus \{x, y, z\}$ where any two vertices u, w are connected by an edge if and only if either $\{x, u, w\} \in \mathcal{B}$ or $\{y, u, w\} \in \mathcal{B}$. An STS($v$) is *perfect* if G_{xy} is a $(v-3)$-cycle for all $\{x, y\} \subseteq V$.

A perfect STS(v) is known only for $v = 7, 9, 25, 33, 79, 139, 367, 811, 1531, 25771, 50923, 61339$, and 69991 [M.J. Grannell, T.S. Griggs, J.P. Murphy, Some new perfect Steiner triple systems, *J. Combin. Des.* **7** (1999), 327–330]. On the other hand, it is known that a perfect STS(v) does not exist for $v = 13, 15$.

In this talk we discuss the computational techniques that were used to establish the nonexistence of a perfect STS(v) for $v = 19, 21$. Incidentally, the nonexistence for $v = 19$ can be obtained through an investigation of the 2591 anti-Pasch STS(19) found in the complete classification [P. Kaski, P.R.J. Östergård, The Steiner triple systems of order 19, *Math. Comp.* **73** (2004), 2075–2092]. For $v = 21$, the algorithm used in the STS(19) classification is reinforced with a disjoint set data structure for pruning partial solutions containing short cycles.